

# Light Water Reactor Sustainability Program

## Markov Process to Evaluate the Value Proposition of a Risk-Informed Predictive Maintenance Strategy



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# **Markov Process to Evaluate the Value Proposition of a Risk-Informed Predictive Maintenance Strategy**

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## ABSTRACT

To achieve high capacity factors, the nuclear fleet has relied on labor-intensive and time-consuming operation and preventive maintenance programs for plant systems. Manually-performed inspection, calibration, testing, and maintenance of plant assets at periodic frequencies, along with the time-based replacement of assets irrespective of condition, have resulted in a costly, *labor-centric business model*.

Fortunately, there are technologies that can eliminate unnecessary preventive maintenance activities by deploying risk-informed predictive maintenance, enabling the transition to a *technology-centric business model*. The technology-centric business model will enable plants to optimize and automate maintenance activities, leading to cost reductions, since labor is a rising cost and technology is a declining cost. The implementation of scalable technologies and methodologies across plant systems and across the nuclear fleet is critical for the successful deployment of a risk-informed predictive maintenance strategy at commercial nuclear power plants.

The work presented in the report is being developed as part of a collaborative research effort between Idaho National Laboratory and Public Service Enterprise Group Nuclear, LLC. This report describes the technical basis using the Markov Process to evaluate the value proposition for the risk-informed predictive maintenance strategy for the circulating water system. Circulating water system plant process data from the Salem and Hope Creek nuclear power plants were utilized to develop a Markov chain risk model and set of equations that allows us to estimate the loss and gain in revenue based on plant availability. The outcomes presented in this report provide the technical basis for an extensive quantitative evaluation of a scalable risk-informed predictive maintenance strategy as part of future research.

## **ACKNOWLEDGEMENTS**

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# CONTENTS

ABSTRACT.....	iii
ACKNOWLEDGEMENTS.....	iv
ACRONYMS.....	ix
1. INTRODUCTION.....	1
1.1 Motivation and Background.....	2
1.2 Report Layout .....	3
2. COST EVALUATION USING MARKOV CHAIN APPROACH .....	3
3. SUMMARY AND PATH FORWARD .....	16
4. REFERENCES .....	16

# FIGURES

Figure 1. Transition from a PM program to a risk-informed PdM program.....	1
Figure 2. Transition diagram for a three-state model.....	3
Figure 3. Transition diagram for a three-state model represented as death-birth process.....	4
Figure 4. A realization of the time-evolution dynamics of a three-state system. The abscissa axis is the time and the ordinate axis is the model's state.....	5
Figure 5. Transition diagram for a two-state model.....	6
Figure 6. A realization of the time-evolution dynamic of a two-state system. ....	6
Figure 7. Transition diagram for the three-state model scaled to handle six CWPs.....	7
Figure 8. Transition diagram for the two-state model scaled to handle six CWPs.....	7
Figure 9. Scaling a three-state model to a two-state model. ....	8
Figure 10. Superposition of events flows for two CWPs.....	9
Figure 11. Six CWPs with a two-maintenance-crews model.....	10
Figure 12. Six CWPs two-maintenance-crews hybrid model. ....	11
Figure 13. System-level mixed-scenario model for derate and trip states. The probability that the system will be returned to $SD$ is $p$ .....	12
Figure 14. Time diagram for calculating the compound failure rate for three pumps. ....	13
Figure 15. Thinning of the random process. ....	14

## TABLES

Table 1. Comparison of the original three-state single-CWP motor model and parameter-scaled three-state single-CWP motor model.....	8
Table 2. Comparison of the original three-state six CWP motor model and parameter-scaled three-state six CWP motor model.....	8
Table 3. Probabilities of different states for six CWPs with a two-maintenance-crews model.....	10
Table 4. Probabilities of different states for the six CWPs two-maintenance-crews hybrid model.....	11
Table 5. Probabilities of different states for the system-level mixed-scenario model with $p = 1$ . ....	15
Table 6. Probabilities of different states for the system-level mixed-scenario model with $p = 0$ . ....	15
Table 7. Probabilities of different states for the system-level mixed-scenario model with $p = 0.5$ . ....	15



## ACRONYMS

CM	corrective maintenance
CWP	circulating water pump
CWS	circulating water system
INL	Idaho National Laboratory
LWR	light water reactor
LWRS	Light Water Reactor Sustainability
NPP	nuclear power plant
O&M	operation and maintenance
PdM	predictive maintenance
PM	preventive maintenance
PSEG	Public Service Enterprise Group
TERMS	Technology-Enable Risk-Informed Maintenance Strategy

# Markov Process to Evaluate the Value Proposition of a Risk-Informed Predictive Maintenance Strategy

## 1. INTRODUCTION

The primary objective of the research presented in this report is to describe the scalability of a deployable risk-informed predictive maintenance (PdM) strategy. As well-constructed PdM approaches, taking advantage of advancements in data analytics, machine learning, artificial intelligence, and visualization, are developed and deployed for an identified plant asset, it is important to ensure the scalability of the approach across plant systems and across the nuclear fleet. This would allow commercial nuclear power plants (NPPs) to achieve a reliable transition from current labor-intensive preventive maintenance (PM) programs to a technology-driven PdM program, as shown in Figure 1, eliminating unnecessary operation and maintenance (O&M) costs. Over the years, the nuclear fleet has relied on labor-intensive and time-consuming PM programs, driving up O&M costs to achieve a high capacity factor.



Figure 1. Transition from a PM program to a risk-informed PdM program.

The value proposition for scalability of the risk-informed PdM strategy presented in this report was developed by Idaho National Laboratory (INL) in collaboration with Public Service Enterprise Group (PSEG) Nuclear, LLC. To develop the initial scalable methods, models, and visualization schemes, the circulating water system (CWS) at PSEG-owned Salem and Hope Creek NPPs was selected as the identified plant asset. The CWS is an important non-safety related system and is omnipresent across the fleet of existing light-water nuclear plants. Traditionally, most of the PdM approaches in the nuclear industry are developed at the component level [1–5]. This approach is not holistic and presents challenges when scaled to the system level. In addition, it prevents nuclear plant sites from harvesting the maximum benefits. Here, benefits could be in terms of automation, optimization of labor and material resources, cost savings, and others. The research approach presented in this report addresses these limitations.

The research and development (R&D) and outcomes reported here are part of the Technology Enabled Risk-Informed Maintenance Strategy (TERMS) Project sponsored by the U.S. Department of Energy’s Light Water Reactor Sustainability (LWRS) program. The LWRS program is an R&D program conducted in close partnership with industry to provide the technical foundations for licensing, managing,

and economically operating the current fleet of NPPs. The LWRS program serves to help the U.S. nuclear industry adopt new technologies and engineering solutions.

Within the LWRS program, the Plant Modernization Pathway conducts targeted R&D to address aging and reliability concerns with legacy instrumentation and control through modernized technologies for the existing U.S. fleet of operating light-water reactors (LWRs) and improved processes for plant operation and power generation. The research goals of the Plant Modernization Pathway are focused on delivering technologies and results that significantly reduce the technical, financial, and regulatory risk of modernization.

To achieve both LWRS program and Plant Modernization Pathway goals [6], a series of pilot projects are underway to develop and demonstrate new technologies that can affect transformative change in the operations and support of nuclear plants. The TERMS pilot project is developing the necessary technologies and methodologies to achieve performance improvement through a transformative transition to PdM.

This research project is designed to help nuclear industry officials understand the benefits of advanced data analytics and risk methodologies in eliminating unnecessary costs associated with labor-intensive time-based PM programs at NPPs. To deliver this message and enable the transition to risk-informed PdM across plant systems and across the nuclear fleet, this report presents scalability R&D activities performed on CWS across two plant sites.

## 1.1 Motivation and Background

Global energy market trends are driven heavily by the abundant reserves of natural gas. As such, there is an immediate need to reduce costs associated with operating and maintaining the current domestic fleet of nuclear plants (99 operating units). Operating in a market selling wholesale electricity for \$22/MWh becomes unsustainable with current nuclear plant O&M costs, which account for at least 66% of the total operating cost. Prices for producing nuclear energy start higher than the market price of electricity, with the nuclear industry average operating cost at approximately \$34/MWh (not specific to PSEG-owned Salem and Hope Creek NPPs) and O&M costs of approximately \$22/MWh. On average, annual O&M costs equate to approximately \$145M per station.

These O&M costs (bundled with the labor-intensive PM program) are right now a major contributor to total operating costs. They involve manually-performed inspection, calibration, testing, and maintenance of plant assets at a periodic frequency, along with the time-based replacement of assets, irrespective of condition. This has resulted in a costly, *labor-centric business model*. Fortunately, there are technologies (advanced sensor, data analytics, and risk assessment methodologies) that can enable the transition to a *technology-centric business model*. The technology-centric business model will result in the significant reduction of PM activities, driving down costs since labor is a rising cost and technology is a declining cost. This transition will also enable nuclear plants to maintain and perhaps even achieve higher capacity factors while still significantly reducing O&M costs.

The challenges facing the industry are clearly understood by regulators, operators, and vendors alike, but there are particular roadblocks that make change difficult to implement. For example, the Nuclear Energy Institute has issued several efficiency bulletins related to reducing the cost of maintenance. The PdM R&D plan [7] laid the foundation for the real-time condition assessment of plant assets. Successful execution of the R&D plan will result in the development of a deployable PdM maintenance program for plant use, thereby enhancing the safety, reliability, and economics of operation.

## 1.2 Report Layout

This report is organized as follows:

- Section 2 discusses the Markov chain process to address scalability and cost evaluation of risk models. Both parametric and non-parametric scaling of two- and three-state Markov models are presented.
- Section 3 summarizes the research accomplishments and presents a path forward in advancing R&D activities.

## 2. COST EVALUATION USING MARKOV CHAIN APPROACH

This section deals with the scalability of continuous Markov models when they are applied to a risk-benefits analysis of operating assets in NPPs. To demonstrate the scalability of the Markov chain model for a particular component, the CWS is selected. Initially, two types of Markov chain models for a single circulating water pump (CWP) and a CWP motor—three-state and two-state models—are applied. The three-state CWP and the CWP motor model are based on the assumption that the maintenance performed on the asset is divided into two categories—corrective and preventive. Corrective maintenance (CM), sometimes called “repair,” is performed when a component fails during operation or during standby (random event). In this situation, performing maintenance is a necessity for returning the component to an operational state. On the other hand, PM is normally performed when a component is operational but requires some service. Often, PM is performed when the component is online; however, PM may require derating the unit. In addition, PMs are periodic and mostly performed at fixed time frequencies. For CM, the component often needs to be taken out of service. While the time intervals between CM events are random, PM is performed more on a scheduled basis; time intervals are more regular but with some variance. The transition diagram for the three-state model is shown in Figure 2.

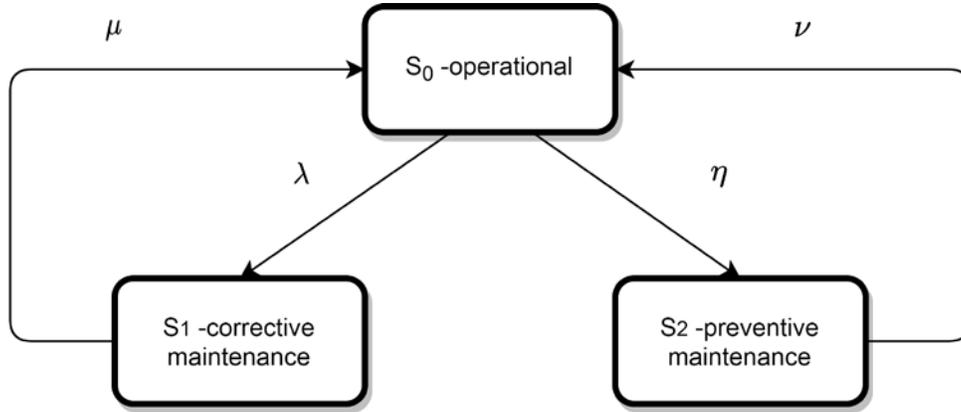


Figure 2. Transition diagram for a three-state model.

The three-state model is completely defined by four parameters:  $\lambda$  represents the failure rate,  $\mu$  represents the CM rate,  $\eta$  represents the PM scheduling rate, and  $\nu$  represents the PM rate and its initial conditions. The system of differential equations, governing the evolution of the three-state model can be written as follows [9]:

$$\frac{dp_0}{dt} = \mu \cdot p_1 - \lambda \cdot p_0 + \nu \cdot p_2 - \eta \cdot p_0$$

$$\frac{dp_1}{dt} = \lambda \cdot p_0 - \mu \cdot p_1 \quad (1)$$

$$\frac{dp_2}{dt} = \eta \cdot p_0 - \nu \cdot p_2$$

$$p_0(0) = 1, p_0(t) + p_1(t) + p_2(t) = 1.$$

where  $p_0$ ,  $p_1$ , and  $p_2$  are probabilities for the component to be in corresponding state.

The transition diagram in Figure 2 can be easily rearranged to represent a death-birth process [8], where the transitions are only possible to the neighboring states as shown in Figure 3.

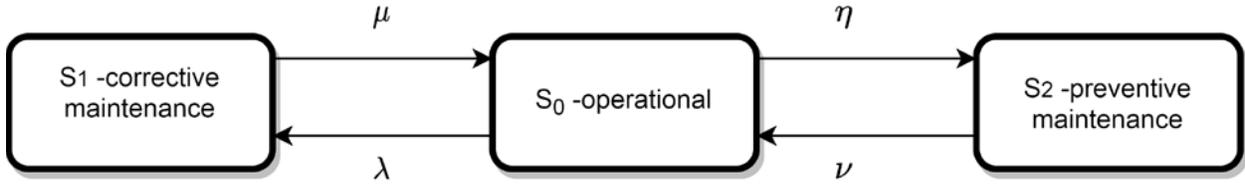


Figure 3. Transition diagram for a three-state model represented as death-birth process.

The advantage of representing a transition diagram as a death-birth process is that in this case, the steady-state probabilities can be calculated analytically [9]. Using the normalization requirement and two of the three equations, the steady-state probabilities can be written as [9]:

$$p_1 = \frac{1}{1 + \frac{\mu}{\lambda} + \frac{\mu \cdot \eta}{\lambda \cdot \nu}}; p_0 = \frac{\mu}{\lambda} p_1; p_2 = \frac{\eta}{\nu} p_1 \quad (2)$$

The steady-state solution is guaranteed to exist for situations when all four parameters are independent of time (i.e., they are constants) [10]. In this report, time-independent parameters are considered. The parameters can be estimated from a motor's operational history and, for the three-state model required, historical data is used to estimate all four parameters. For details, refer to [4]. Given the parameters, the probabilities of the asset to be in a specific state (i.e. being operational or being under PM or CM) are estimated [4]. The parameters can be estimated from operational data, as shown in Figure 4, where  $\tau_i^o$  are time intervals when the CWP is fully operational,  $\tau_i^r$  are time intervals when the CWP or the CWP motor is undergoing CM,  $\tau_i^m$  are time intervals when the CWP or the CWP motor is in PM, and  $\tau_i^s$  is the time interval between scheduled PMs. Having obtained these time intervals from operational data, the four parameters can be estimated using the procedure described in [4]. For Salem Unit 1, the following values were obtained,  $\lambda = 3.6 \times 10^{-4}$ ,  $\mu = 1.8 \times 10^{-2}$ ,  $\eta = 6.4 \times 10^{-5}$ , and  $\nu = 7.5 \times 10^{-2}$ . The steady-state probabilities calculated using Equation (2) are  $p_0 = 0.97952$ ,  $p_1 = 0.019645$ , and  $p_2 = 0.00083455$ .

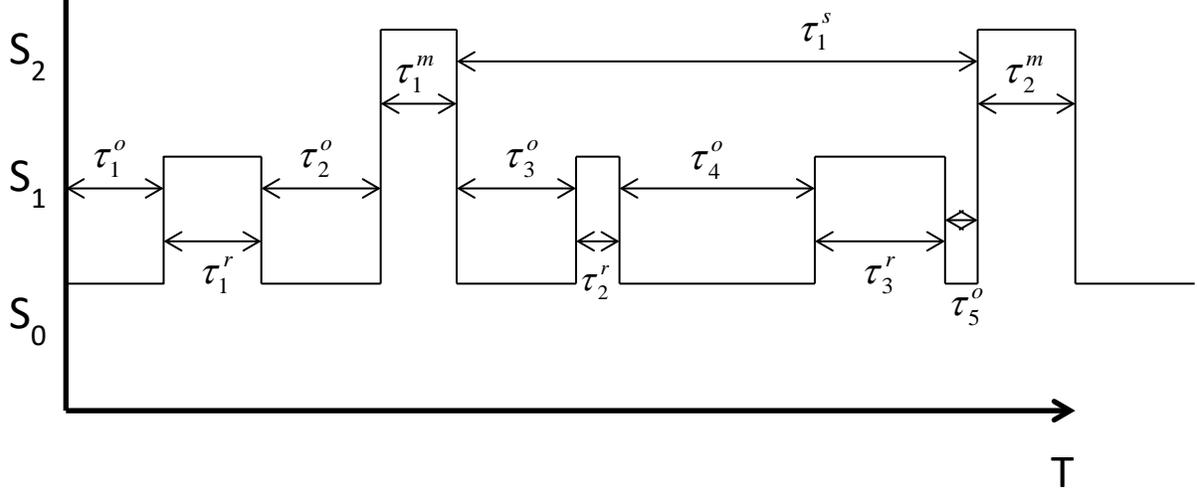


Figure 4. A realization of the time-evolution dynamics of a three-state system. The abscissa axis is the time and the ordinate axis is the model's state.

Given the four parameters, the following variables can be computed.

$$\begin{aligned}
 MTBF &= \frac{1}{N_o} \sum_{i=1}^{N_o} \tau_i^o \text{ and Failure rate } \lambda = \frac{1}{MTBF} \text{ (per unit of time)} \\
 MCMT &= \frac{1}{N_c} \sum_{i=1}^{N_c} \tau_i^r \text{ and Corrective maintenance rate } \mu = \frac{1}{MCMT} \text{ (per unit of time)} \quad (3) \\
 MPMT &= \frac{1}{N_p} \sum_{i=1}^{N_p} \tau_i^m \text{ and Preventive maintenance rate } \nu = \frac{1}{MPMT} \text{ (per unit of time)} \\
 MTSM &= \frac{1}{N_{PS}} \sum_{i=1}^{N_{PS}} \tau_i^s \text{ and Preventive maintenance scheduling rate } \eta = \frac{1}{MTSM} \text{ (per unit of time)}
 \end{aligned}$$

where  $MTBF$  represents the mean time between failures,  $MCMT$  represents the mean CM time,  $MPMT$  represents the mean PM time, and  $MTSM$  represents the mean time between scheduled PMs.  $N_o$  is the number of times the CWP was in the  $S_0$  state (here  $N_o = 5$ ),  $N_c$  is the number of times the CWP was in the  $S_1$  state (here  $N_c = 3$ ),  $N_p$  is the number of times the CWP was in the  $S_2$  state (here  $N_p = 2$ ), and  $N_{PS}$  is the number of times the CWP entered the  $S_2$  state (here  $N_{PS} = 1$ ). The steady-state probabilities have important practical interpretations as average relative percentages of time the system spends in particular states. This interpretation, for example, allows for calculating the hourly profit for a particular asset given hourly rates, as shown in Equation (4).

$$Profit = Revenue - Expense = Hourly\ rate \cdot p_0 - Expense\ hourly\ rate \cdot p_1 \quad (4)$$

In a more general form, the profit equation can be written for a system that can be in  $N$  number of states as:

$$Profit = \sum_{i=1}^N Cost_i \cdot p_i \quad (5)$$

where  $Cost_i$  is cost of being in the  $i - th$  state, which is calculated by considering the revenue and expense of being in the  $i - th$  state. Notice,  $Cost_i$  could be positive or negative depending on whether the system is making or losing money by being in a corresponding state.

Another commonly used Markov chain model is a two-state model where the two types of maintenance are merged into one state, as shown in Figure 5.

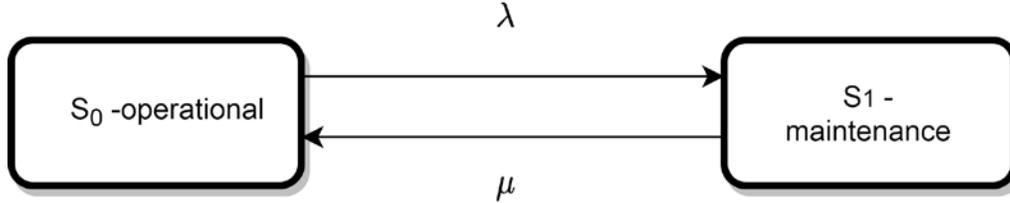


Figure 5. Transition diagram for a two-state model.

The two-state model is described by the system of two differential equations, along with initial conditions:

$$\begin{aligned} \frac{dp_0}{dt} &= \mu \cdot p_1 - \lambda \cdot p_0 \\ \frac{dp_1}{dt} &= \lambda \cdot p_0 - \mu \cdot p_1 \end{aligned} \quad (6)$$

$$p_0(0) = 1, p_0(t) + p_1(t) = 1.$$

Notice that the two-state model represents the simplest type of death-birth process with only two states. Using the normalization requirement and one of the two equations, the steady-state solutions for the two-state system can be expressed as:

$$p_0 = \frac{\mu}{\mu + \lambda}; p_1 = \frac{\lambda}{\mu + \lambda} \quad (7)$$

In contrast to the three-state model, the two-state model has only two parameters,  $\lambda$  and  $\mu$ . These parameters can be estimated from operational data as shown in Figure 6, and following the procedure described in [4].

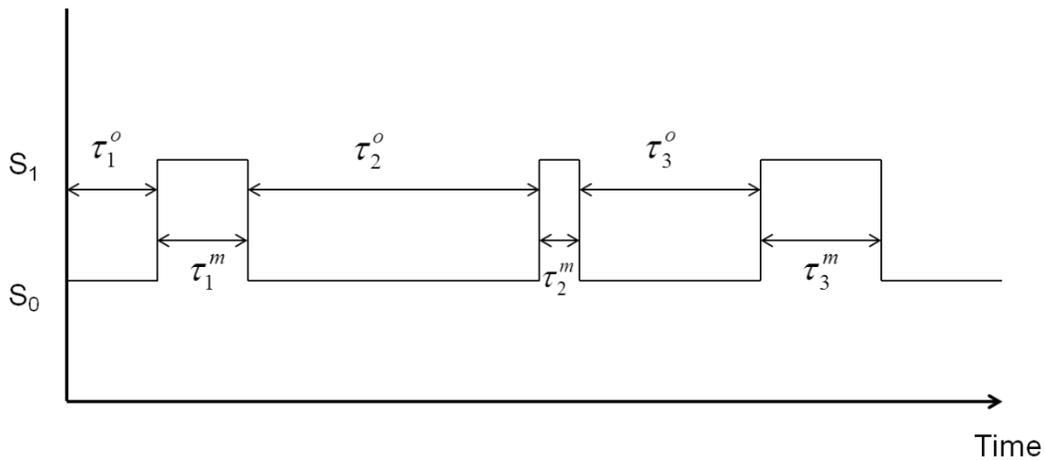


Figure 6. A realization of the time-evolution dynamic of a two-state system.

Here  $\tau_i^m$  are the time intervals when the pump is undergoing PM or CM. Note, that the  $\tau_i^m$  described here is different from the one in Figure 4. After estimating these time intervals from operational data, the *MTBF* can be obtained using its expression in Equation (3), while the mean maintenance time, denoted as *MMT*, is computed as,

$$MMT = \frac{1}{N} \sum_{i=1}^N \tau_i^m \text{ and Maintenance rate } \mu = \frac{1}{MMT} \text{ (per unit of time)} \quad (8)$$

where  $N$  is the number of times the CWP is in the  $S_1$  state.

So far, three- and two-state models are used to describe the state of the CWP and the CWP motor of the CWS. Both models can be scaled to a system level by rescaling the parameters. Scalability via a parameter estimation approach was considered first and presented here. For example, Figure 7 and Figure 8 show three- and two-state models rescaled to handle a CWS with six CWPs and CWP motors.

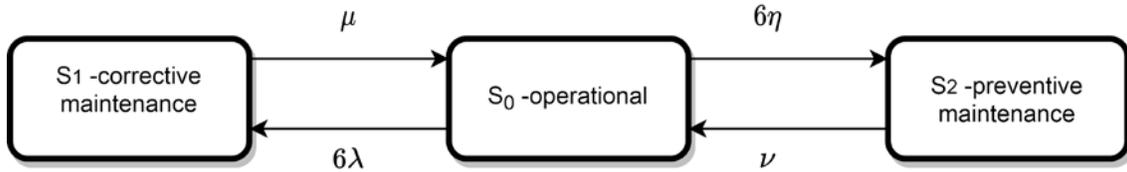


Figure 7. Transition diagram for the three-state model scaled to handle six CWPs.

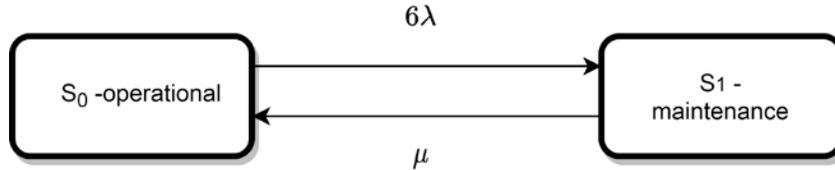


Figure 8. Transition diagram for the two-state model scaled to handle six CWPs.

Notice that the transition rates from the operational state are multiplied by a factor of six as any of the six CWP motors can fail. The interpretation of the model's states after scaling is also different from a single-CWP motor model. For a single-CWP motor model, the operational state means that the CWP motor is fully operational, while the maintenance state means that the CWP motor is down and under maintenance. In the scaled models (Figure 7 and Figure 8), operational state means that all CWP motors are operational. In the case of Salem NPP, all six CWP motors are operational, and, in case of the Hope Creek NPP, all four CWP motors are operational. Similarly, for scaled models, the maintenance state means that at least one CWP motor is down and under maintenance.

While the scaled three- and two-state system-level models can handle systems with different numbers of components, they cannot be used to reflect the derate in gross load and plant trips when a certain number of CWP motors are unavailable. The models considered so far are effectively binary models, as each state is either “operational” or “maintenance.” Notice that in Figure 7 and Figure 8, the maintenance rates remained the same as in the single-motor models (Figure 4 and Figure 5), as only one maintenance crew is assumed.

Analysis of Figure 2 and Figure 3 reveals that the three-state model can be transformed or scaled to a two-state model by combining PM and CM into a single state and adding the corresponding rates, as shown in Figure 9.

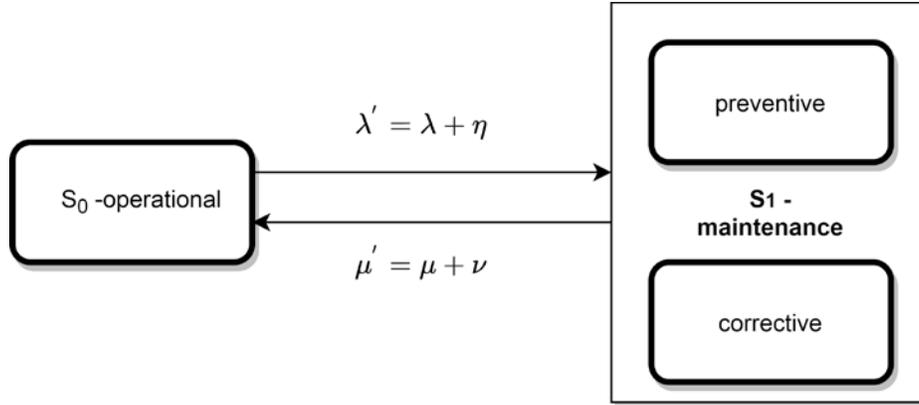


Figure 9. Scaling a three-state model to a two-state model.

The summation of the rates requires an assumption that the underlying point processes are simple Poisson processes with interarrival time intervals distributed exponentially [1]. This assumption may not be correct for parameter  $\eta$  as it is a preventive scheduling rate, which tends to be performed at predefined time intervals. Using a single-CWP motor failure and maintenance rates for the three-state system, which are  $\lambda = 6.01 \times 10^{-5}$ ,  $\mu = 1.8 \times 10^{-2}$ ,  $\eta = 1.06 \times 10^{-5}$ , and  $\nu = 7.5 \times 10^{-2}$ ; and revised failure and maintenance rates for the scaled three-state system, which are  $\lambda' = 7.08 \times 10^{-5}$  and  $\mu' = 2.03 \times 10^{-5}$ , the states' probabilities were calculated. For the comparison of two models, the parameter  $p_0$  is interpreted as the probability of being in an operation state, and parameter  $p_1$  is interpreted as the probability of being in a maintenance state. Table 1 and Table 2 shows these two parameters for the three-state model and scaled three-state model for a single pump along with profit information. For the profit calculations, the following dollar values were used: \$34 per Megawatt hour (MWh) was used as the revenue hourly rate and, for both CM and PM, the hourly rates were assumed to be \$100/MWh, which included labor and parts costs. In addition, for both types of maintenance, it was assumed that the unit was either offline or derated and thus an additional cost of \$34/MWh was added bringing the hourly maintenance cost to \$134/MWh. Equation (5) was used to calculate hourly profit.

Table 1. Comparison of the original three-state single-CWP motor model and parameter-scaled three-state single-CWP motor model.

	Three-state model	Scaled three-state model
P(operational)	0.99653	0.99652
P(maintenance)	0.0035	0.0035
Profit, \$/hour	33.417	33.417

For the six-pump CWS systems depicted in Figure 7 and Figure 8, the corresponding results are presented in Table 2.

Table 2. Comparison of the original three-state six CWP motor model and parameter-scaled three-state six CWP motor model.

	Three-state model	Scaled three-state model
P(operational)	0.97952	0.9795
P(maintenance)	0.0205	0.0205
Profit, \$/hour	30.6	30.6

For the results presented in Table 2, the following parameters have been used:  $\lambda = 3.6 \times 10^{-4}$ ,  $\mu = 6.4 \times 10^{-5}$ ,  $\eta = 6.4 \times 10^{-5}$ , and  $\nu = 7.5 \times 10^{-2}$  for the original three-state model and  $\lambda' = 2.03 \times 10^{-5}$  and  $\mu' = 2.03 \times 10^{-5}$  for the scaled three-state six CWP motor model.

As we can see from Table 1 and Table 2, the two models produced identical results, with the six-pump models having lower probabilities of being operational and higher probabilities of being in maintenance. For the three-state model, the probability of being in a maintenance state is the sum of the probabilities of being in a CM state and a PM state. These results are intuitively correct because the probability that a single-CWP motor will go down is lower than the probability that out of six CWP motors at least one will be down. These results confirm the scalability of the three-state and two-state models with respect to the parameters superposition and number of states. Also, notice that due to the lower probability of a single-motor failure, the single-motor model shows a higher hourly profit than the six-motor model.

The calculation of compound rates  $\lambda'$  and  $\mu'$  to scale from a three-state model to a two-state model can either be performed by adding the corresponding three-state model rates, as shown in Figure 9, or it can be performed directly from the data, as shown Figure 10 for a two-pump case. In this case, the resulting flow will have higher rates than those of individual processes but may not be an arithmetic sum of their individual rates. The time of event occurrences in Figure 10 is designated with letter  $E_{ij}$ , where  $i$  is the motor index and  $j$  is the event index.

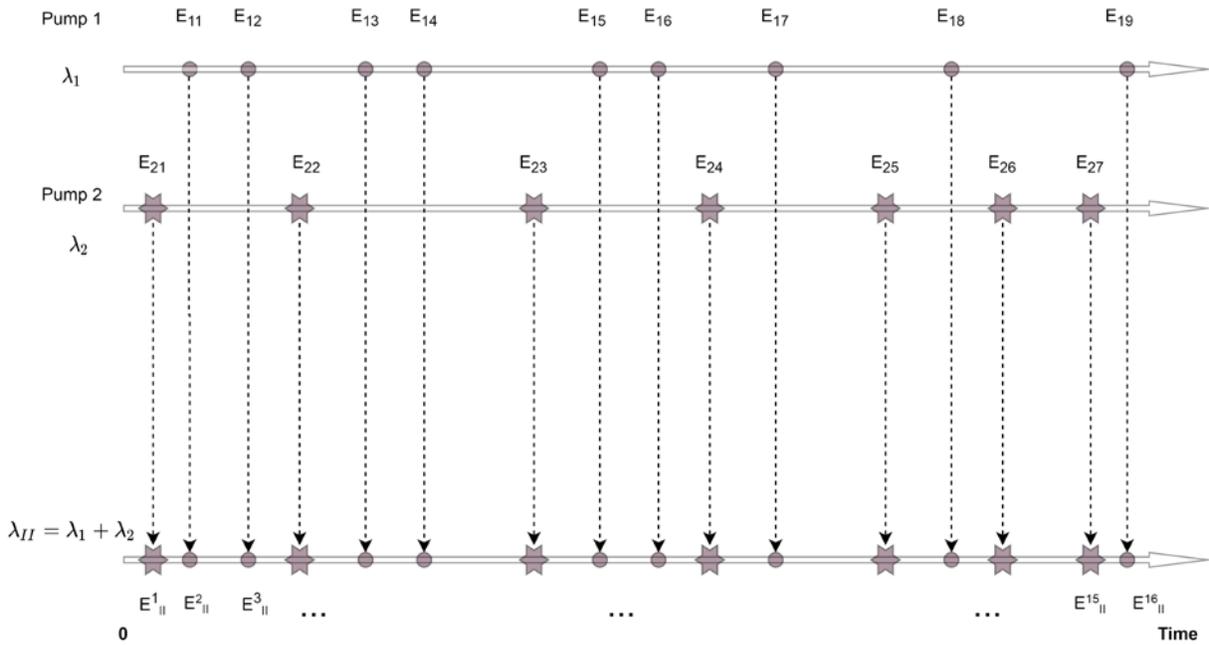


Figure 10. Superposition of events flows for two CWPs.

The scalability results presented so far only dealt with models describing the component or the whole system either in a fully functional state or in a maintenance state. This approach does not account for the impact of CWP unavailability on a plant derate or trip. The scalable models explained here will handle the derate and trip situations. Defining such models also requires a slightly different approach to the definition of the model's states and transition rates. For a CWS with six CWPs, the most basic model capable of handling derated states for both risk and cost calculations are shown in Figure 11.

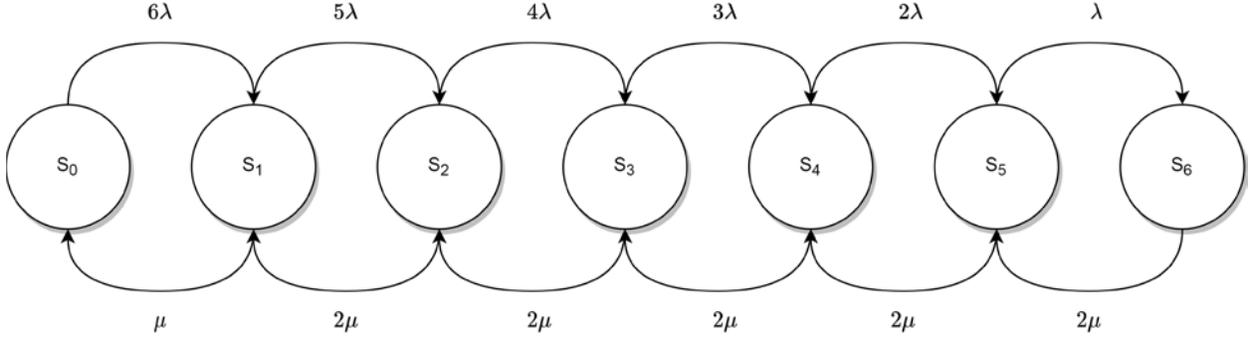


Figure 11. Six CWPs with a two-maintenance-crews model.

This model is a full six-motor model that handles failures of each individual pump as well as derated and tripped states. The parameter  $\lambda$  in this model is a single-pump failure rate, and  $\mu$  is a single-crew maintenance rate. The model has seven states, with  $S_0$  corresponding to a fully operational state with no pumps failed,  $S_1$  corresponding to a state with one pump failed and undergoing maintenance, and five pumps operational, and so on. The final  $S_6$  state corresponds to all six pumps undergoing maintenance. Notice that transition failure rates for this model are products of the single-pump failures rates,  $\lambda$ , and the number of operational pumps. This accounts for changing failure rates for a different number of pumps. The model assumes that only two maintenance crews are available, so, if more than one pump failed, the maintenance rate cannot be higher than  $2\mu$ . The six-pumps model is a death-birth model with seven states. Its steady-state probabilities can be calculated analytically according to the following formula:

$$p_0 = \frac{1}{1 + \sum_{i=1}^6 \prod_{k=1}^i \frac{6!}{(6-k)!} \cdot \frac{\lambda^k}{\mu^k} \cdot \frac{1}{2^{k-1}}} \quad (9)$$

with

$$p_i = p_0 \cdot \prod_{k=1}^i \frac{6!}{(6-k)!} \cdot \frac{\lambda^k}{\mu^k} \cdot \frac{1}{2^{k-1}}, i = 1, \dots, 6 \quad (10)$$

where  $p_i$  is the probability that the model is in the  $i$ -th state. By setting the single-motor failure rate  $\lambda = 7.08 \times 10^{-5}$  and single-crew maintenance rate to  $\mu = 2.03 \times 10^{-5}$ , the following state probabilities can be obtained as shown in Table 3.

Table 3. Probabilities of different states for six CWPs with a two-maintenance-crews model.

State	$S_0$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	Profit, \$/hour
P(State)	0.9793	0.0205	0.0002	$1.2472 \cdot 10^{-6}$	$6.5267 \cdot 10^{-9}$	$2.2769 \cdot 10^{-11}$	$3.9714 \cdot 10^{-14}$	31.9

The profit for the six-motors with the two-maintenance-crews model shows a bit higher profit than the models in Table 2; however, the difference is not significant. For the six-motor model, it was assumed that the loss of a single pump causes a 5% loss in revenue due to deration, two pumps—10% loss, and three pumps—20% loss.

Despite the fact that the six-pump model is a death-birth model (i.e., the transitions are only possible between neighboring states), it handles the situation of several motors being down at the same time, with two pumps being maintained at the same time. Also, the model only requires single-pump and single-crew rates to handle such situations. A comparison of the state probabilities of this model with the state probabilities of the system-level three-state and two-state models, shown in Table 2, reveals that the probabilities of operational states are very similar. The probability of the six-pump model being in maintenance is calculated as a sum of the probabilities of all states. But  $S_1$  is 0.0207, which is similar to

the maintenance probabilities of three- and two-state system-level models. However, in contrast to those models, the six-pump model can be used to calculate the probabilities of derated states and tripped states separately. Knowing these probabilities is important for risk-benefits calculations.

The six-pump model can be simplified if some states can be merged into a single state. For example, if a unit is tripped when more than three pumps are down, the model can be represented as shown in Figure 12.

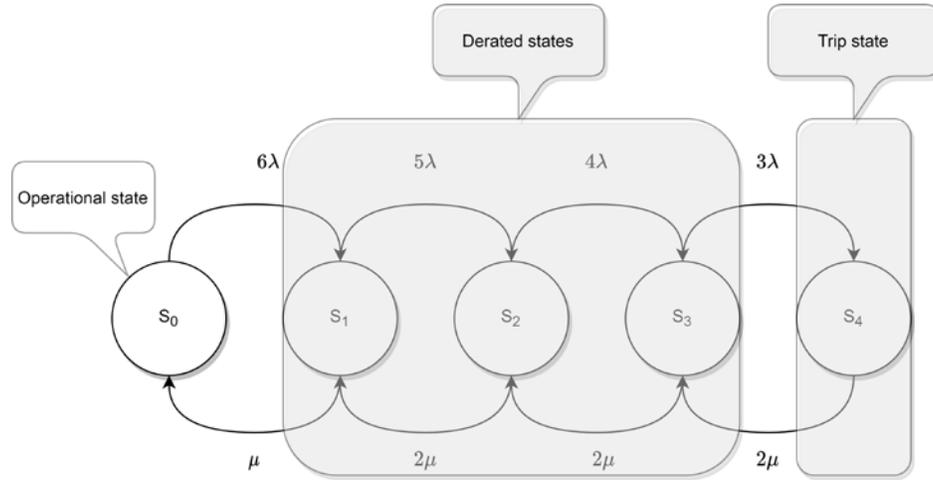


Figure 12. Six CWPs two-maintenance-crews hybrid model.

This model can be called a hybrid model as it handles the failures of individual pumps as well as system-level failures through a single state assigned to the occurrence of a trip. The state probabilities for this model are shown in Table 4.

Table 4. Probabilities of different states for the six CWPs two-maintenance-crews hybrid model.

State	$S_0$	$S_1$	$S_2$	$S_3$	$S_4$	Profit, \$/hour
P(state)	0.9793	0.0205	0.0002	$1.2472 \cdot 10^{-6}$	$6.5267 \cdot 10^{-9}$	31.9

The profit for this model is identical to the full six-motor model as the steady-state probabilities are equal. The losses due to derated states were the same as for the full six-motor model.

A comparison of Table 3 and Table 4 reveals that the hybrid model produces virtually identical results to the six-motor-pumps model with the exception of states  $S_5$  and  $S_6$ , which are not present in the hybrid model. However, the probabilities of those states are orders of magnitude lower than the probability of  $S_4$  and have virtually no influence on the overall performance of the model.

A natural generalization of the hybrid model can be obtained by merging states  $S_1$ ,  $S_2$ , and  $S_3$  into a single state  $S_D$ , which is referred as the derated state. This model is shown in Figure 13. In this model,  $S_0$  is a fully operational state with all six pumps available;  $S_D$  is a derated state with one, two, or three pumps under maintenance; and  $S_T$  is a trip state with more than three pumps under maintenance. Notice that, for this model, an additional transition rate directly from  $S_T$  to  $S_0$  must be used to account for the possibility of different maintenance scenarios at different utilities. Parameter  $p$  is the probability that a utility will choose to go online as soon three pumps are available, while  $1 - p$  is the probability that the utility will wait until all six pumps are available before going online. Both scenarios are possible, and this model provides an additional scalability to utilities' maintenance policies. While the second option delays going

online, it provides a safety margin in case one of the pumps goes down again. Due to the additional edge connecting  $S_T$  and  $S_0$ , the mixed-scenario model is not a death-birth model, and no analytical solution is available for steady-state probabilities. Instead, the following system of differential equations needs to be solved along with normalization conditions:

$$\frac{dp_D}{dt} = \lambda_d \cdot p_o + (\mu_T \cdot p) \cdot p_T - \mu_d \cdot p_D - \lambda_T \cdot p_D \quad (11)$$

$$\frac{dp_o}{dt} = \mu_d \cdot p_D + (\mu_T \cdot (1 - p)) \cdot p_T - \lambda_D \cdot p_o \quad (12)$$

$$\frac{dp_T}{dt} = \lambda_T \cdot p_D - (\mu_T \cdot p) \cdot p_T - (\mu_T \cdot (1 - p)) \cdot p_T \quad (13)$$

where  $p_D$ ,  $p_o$ , and  $p_T$  are the probabilities of corresponding states  $S_D$ ,  $S_0$ , and  $S_T$ .

The transition rates are shown next to the arrows. Each rate is a compound rate, and we shall consider how they can be evaluated. First, we need to state the underlying assumptions for the flow of events in Markov chain modeling. The most important assumption is that the event-flows are assumed to be ordinary, stationary, and memoryless. Stationarity implies that the rate is not changing over time, memorylessness means that future events are not dependent on previous events, and the flow is called “ordinary” if two events cannot happen at the same time [11].

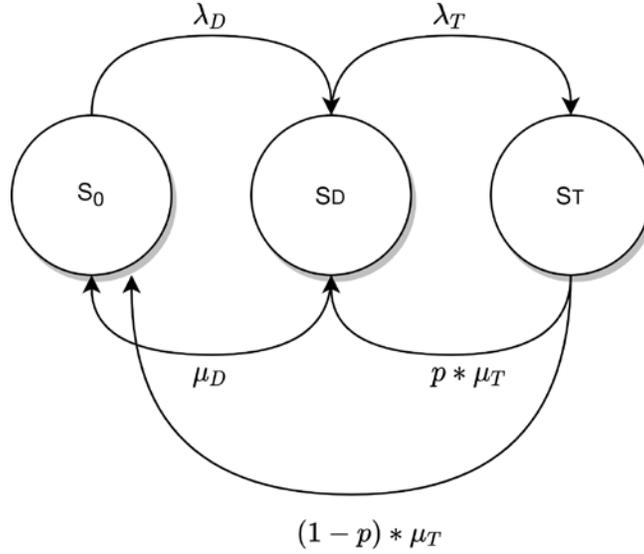


Figure 13. System-level mixed-scenario model for derate and trip states. The probability that the system will be returned to  $S_D$  is  $p$ .

In Figure 13, the rate  $\lambda_D$  is a compound rate of transferring from an operational state to a derated state. Figure 14 demonstrates the compounding diagram for evaluating  $\lambda_D$ . The diagram needs to be analyzed from top to bottom and from left to right. The top three timelines represent the flow of failures for three different pumps. Each pump fails with a corresponding rate,  $\lambda_i$ , at random time instances denoted as  $E_j$ , and the maintenance duration is represented as  $\tau_{ij}$ . For three pumps, there might be a number of combinations of how they can fail. The simplest one is a single-pump failure, represented by a corresponding,  $\lambda_i$ . More complex is a failure of two pumps. By two-pump failure, we do not mean the simultaneous failure of two pumps at the same time but rather when a second pump fails while one motor is already down and undergoing maintenance, as shown in Figure 14. The three-pump failure is the extension of a previous situation where the third motor fails while two are already in maintenance. The

compounding event-flows for two- and three-pump failures are shown in the two bottom timelines. For a three-motor configuration, the compounding failure rate will be:

$$\lambda_D = \sum_{i=1}^3 \lambda_i + \lambda_{II} + \lambda_{III} \quad (14)$$

Assuming that the failure of four or more motors is an extremely unlikely event for a six-motor configuration, the compound rate can be written as:

$$\lambda_D = \sum_{i=1}^6 \lambda_i + \lambda_{II} + \lambda_{III} \quad (15)$$

Since the failure of more than three motors is extremely unlikely, the system cannot move to the trip state directly from the operation state. The transfer to the trip state is only possible via the derated state.

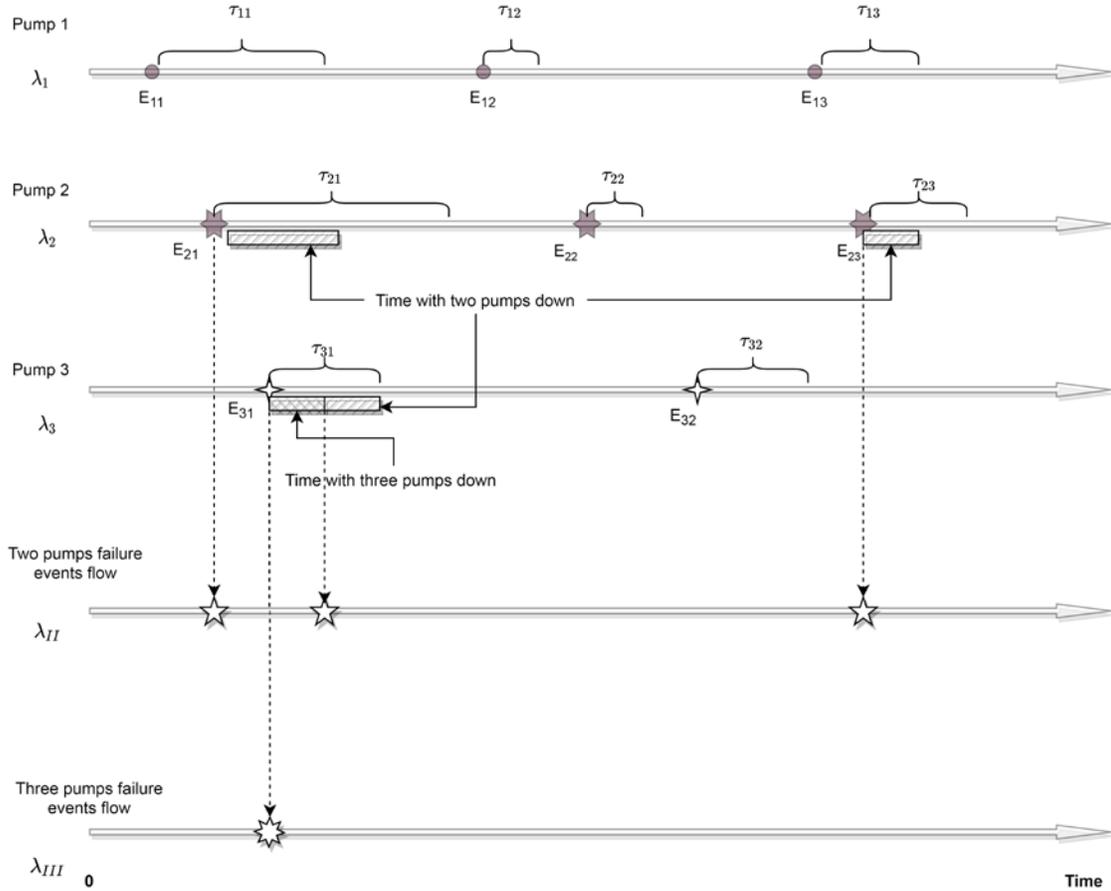


Figure 14. Time diagram for calculating the compound failure rate for three pumps.

Once in a derated state, the system can move back to an operational state with a maximum rate of  $\mu_D = 2\mu$ , where  $\mu$  is the maintenance rate for a single pump and two maintenance crews are assumed. Also, from a derated state, the system can move to the trip state with a compound rate similar to  $\lambda_D$ . Since when in a fully derated state (three motors down) only three motors are left, the compound transition rate will be:

$$\lambda_T = \sum_{i=1}^3 \lambda_i + \lambda_{II} + \lambda_{III} \quad (16)$$

While in a trip state, the model can move to either a derated state or a fully operational state when all six CWP's are operating, as shown in Figure 13. The probability  $p$  can be determined from the utilities' operational data. If failure data for all pumps are not available, the double- and triple-pump failure rates

can be determined from a single-motor failure rate through the process of random process thinning, as illustrated in Figure 15.

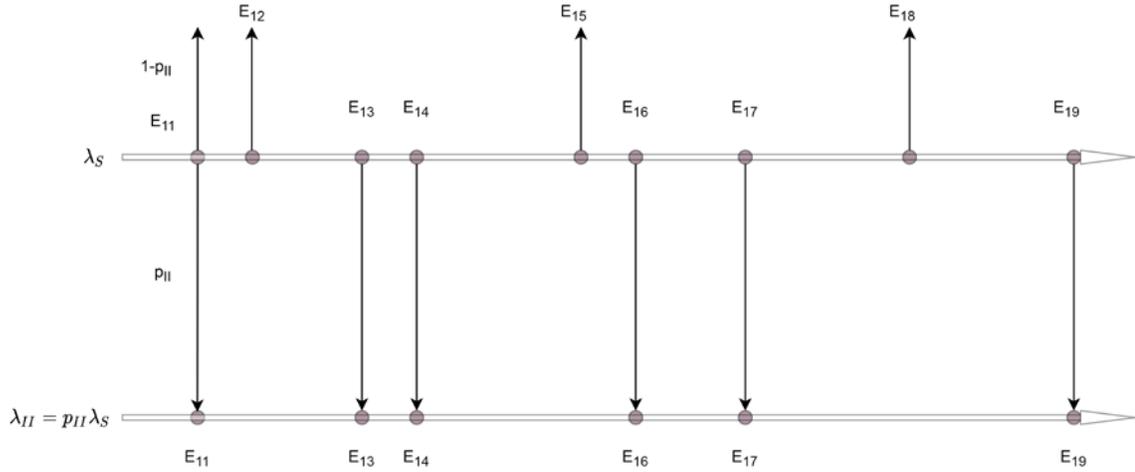


Figure 15. Thinning of the random process.

Assuming the single-pump failure rate is  $\lambda_S$ , we can calculate  $\lambda_{II}$  and  $\lambda_{III}$  as

$$\lambda_{II} = p_{II} \cdot \lambda_S \quad (17)$$

$$\lambda_{III} = p_{III} \cdot \lambda_S \quad (18)$$

where  $p_{II}$  is the thinning probability for a two-pump failure rate and  $p_{III}$  is the thinning probability for the triple-pump failure rate. The process of thinning takes a single-motor failure process, as in our case, and goes over each event in that process. The thinning process either keeps the event with probability  $p_{II}$  or removes it with probability  $1 - p_{II}$  to obtain a double-pump failure rate. The same operation is performed to obtain triple-pump failure rates; however, in this case, the thinning probability is  $p_{III}$ . As a result, thinner processes are produced, which represent double- and triple-pump failure rates. Assuming both processes are Poisson, the relationship between  $\lambda$ 's (i.e.,  $\lambda_i$ ,  $\lambda_{II}$ , and  $\lambda_{III}$ ) can be expressed by Equations (15) and (16). The key parameters for this operation are the thinning probabilities  $p_{II}$  and  $p_{III}$ , which can be estimated from operational data. There is actually no need to thin the process—we are just manipulating  $\lambda$ 's. Having single-, double-, and triple-failure rates, we can use Equations (13) and (14) to obtain  $\lambda_D$  and  $\lambda_T$ , respectively, and, assuming  $\mu_D = 1.5\mu$  and  $\mu_T = 2\mu$ , we can calculate the probabilities of different states for the mixed-scenario model for different maintenance scenarios.

For this report, we used,  $p = 1$ ,  $p = 0$ , and  $p = 0.5$ . For single-, double-, and triple-pump failure rates, we used  $\lambda = 7.08 \times 10^{-5}$ ,  $\lambda_{II} = 2.99 \times 10^{-4}$ , and  $\lambda_{III} = 3.31 \times 10^{-5}$ . The double- and triple-failure rates were obtained through the thinning process of a single-pump failure rate and by multiplying the thinned rate by 15 for the double rate and by 20 for the triple rate to account for the number of combinations of two or three pumps out of six that can fail. Steady-state probabilities are presented in Table 5, Table 6, and Table 7 for the different values of  $p$ . Profit calculations for the models presented in Table 5, Table 6, and Table 7 were modified to reflect the fact that derated and tripped states in this case are compound states, and, when being, for example, in derated state, we do not have information on how many pumps are down—one, two, or three. To deal with this problem, the expenses in derated and tripped states were calculated as:

$$E_D = \sum_{i=1}^3 \pi_i \cdot 100 \quad (19)$$

where  $E_D$  represents the expense of being a derated state and  $\pi_i$  is the steady-state probability of being in the  $i - th$  derated states. While those probabilities are not available for this model, they can be obtained from the full six-motor model in Table 3. Similar calculations can be applied to estimate revenue in the derated state. Notice that revenue in the tripped state is zero.

Table 5. Probabilities of different states for the system-level mixed-scenario model with  $p = 1$ .

State	$S_0$	$S_D$	$S_T$	Profit, \$/hour
P(state)	0.9754	0.0243	3.2501e-04	31.5

Table 6. Probabilities of different states for the system-level mixed-scenario model with  $p = 0$ .

State	$S_0$	$S_D$	$S_T$	Profit, \$/hour
P(state)	0.9758	0.0238	3.1921e-04	31.5

Table 7. Probabilities of different states for the system-level mixed-scenario model with  $p = 0.5$ .

State	$S_0$	$S_D$	$S_T$	Profit, \$/hour
P(state)	0.9756	0.0240	3.2227e-04	31.5

Table 5 shows results for  $p = 1$  (i.e., the model can be returned to a fully functional state via the derated state). This scenario may arise if a utility wants to go online as soon as possible. Notice that the probability of a fully operational state is practically identical to the similar probability of all other models, such as system-level two- and three-state models, a six-motor model, and a hybrid model. The probability of a derated state is also similar for all of these models. While the probability of being in a tripped state is small for all of these models, the probability is higher for the mixed-scenario model. The mixed-scenario model will require more research. It also should be noted that the hourly profit is identical for all three models shown in Table 5, Table 6, and Table 7.

Table 6 shows results for  $p = 0$  (i.e., the model is returned to the fully operational state only directly from a tripped state). This scenario is possible if a utility wants to have a safety margin before returning to an online status. As can be seen from Table 6, in this case, the probability of a fully operational state is slightly higher than for  $p = 1$ , as expected, because the model cannot return to a derated state directly from a tripped state. While both of these scenarios are plausible scenarios, the most likely scenario is when a mixture of these two approaches is used by a utility. That is, sometimes a utility chooses to get online as soon as possible, and sometimes it is willing to delay a return to operations in order to guarantee a safety margin. This mixed scenario can be represented by setting  $p = 0.5$  (i.e., assuming that a utility exercises one of these possibilities equally). The estimate for  $p$  can be obtained from operational experience data. Table 7 shows results for  $p = 0.5$ . The steady-state probabilities are similar to two previous cases; however, the steady-state probabilities for  $p = 0.5$  are effectively arithmetic means of other two cases. For example,  $Probability(S_0|p = 0.5) = 0.9756$ , which is equal to  $(0.9754+0.9758)/2$ -the arithmetic mean of steady-state fully functional probabilities for the other two states.

In summary, we considered the scalability of a continuous-time Markov chain applied to the reliability analysis of a CWS. A number of different models have been analyzed and researched. Specifically, three-state and two-state single-pump and system-level models were investigated. These models were found capable of scaling upward and downward through parameter superposition. Similarly, a three-state model can be scaled to a two-state model, either for a single pump or for a system. The steady-state probabilities are similar to all models demonstrating consistency and interchangeability.

Also, a motor-level model has been analyzed in terms of scalability and was demonstrated as scalable to a single-motor as well as to a system-level. The steady-state probabilities are very similar for that model regardless of whether it is motor-level or system-level. The disadvantage of component-level

models is that, for a large number of components, they may become very complicated and convoluted. This requires system-level models such as the mixed-scenario models considered above. Such models can be scaled and adapted to different systems and different numbers of components.

### 3. SUMMARY AND PATH FORWARD

This report presented different Markov chain models developed by INL in collaboration with PSEG Nuclear, LLC-owned Salem and Hope Creek NPPs. The developed models provide the technical basis to evaluate the value proposition of a risk-informed PdM strategy. Two- and three-state Markov chain risk models developed at a component-level to a system-level and even to the plant-level were demonstrated on the Salem CWS. A component-level two-state Markov chain model that can capture system-level performance and a three-state system-level Markov chain model that can capture plant-level performance were developed. Both two- and three-state Markov chain models were used to estimate the profit based on the time CWP spends in a particular state.

The path forward for this research project for the next year includes performing R&D in collaboration with PSEG Nuclear, LLC and PKMJ Technical Services in performing rigorous evaluations and validations of two- and three-state Markov chain models to quantify the cost-effectiveness of deploying scalable risk-informed PdM strategy. This would enable the cost-benefit analysis of the risk-informed PdM strategy across the nuclear fleet.

### 4. REFERENCES

1. McJunkin, T., V. Agarwal, N. J. Lybeck, and C. Rasmussen 2015. "Online Monitoring of Induction Motors," INL/EXT-15-36681, Idaho National Laboratory.
2. Agarwal, V., N. J. Lybeck, and B. T. Pham. 2014. "Diagnostic and Prognostic Models for Generator Step-up Transformers," INL/EXT-14-33124, Idaho National Laboratory.
3. Agarwal, V., N. J. Lybeck, L. C. Matarica, and B. T. Pham. 2013. "Demonstration of Online Monitoring for Generator Step-up Transformers and Emergency Diesel Generators," INL/EXT-13-30155, Idaho National Laboratory.
4. Goss, N., et al. 2020. "Integrated Risk-Informed Condition Based Maintenance Capability and Automated Platform: Technical Report 1," PKM-DOC-20-0013, PKMJ Technical Services.
5. Agarwal, V., et al. 2019. "Deployable Predictive Maintenance Strategy Based on Models Developed to Monitor Circulating Water System at the Salem Nuclear Power Plant," INL/LTD-19-55637, Idaho National Laboratory.
6. Idaho National Laboratory. 2019. "Light Water Reactor Sustainability Program, Integrated Program Plan," INL/EXT-11-23452, Rev. 8, Idaho National Laboratory.
7. Agarwal, V. 2018. "Risk-Informed Condition-Based Maintenance Strategy: Research and Development Plan," INL/LTD-18-51448, Idaho National Laboratory.
8. Feller W. 1968. *An Introduction to Probability Theory and Its Applications*, Vol. 1 of *Wiley mathematical statistics series, 3rd Edition*. Hoboken, NJ: Wiley.
9. Winston, W. L. 1991. *Operations Research: Applications and Algorithms*. 2nd ed. Boston: PWS-Kent Publishing.
10. Kleinrock, L. 1975. *Theory*. Vol. I of *Queueing Systems*. Hoboken, NJ: Wiley.
11. Nelson, R. 1995. *Probability, Stochastic Processes, and Queueing Theory*. New York: Springer-Verlag.